ECE-311 (ECE, NDSU)
Lab 3 – Experiment
Impedance 1: Phasor relationships

1. Objective

The response of an electrical network to a sinusoidal input forcing function is an extremely important characteristic. This lab investigates the amplitude and phase relationships between voltages and currents in electrical networks driven by sinusoidal sources. In particular, the concepts of phasors and impedance are examined.

2. Introduction

First, you should refresh your memory on the concepts of phasors and impedance. These concepts were covered in Circuit Analysis I. Please revisit your lecture notes and read chapter 10 from textbook at this time.

Measuring Impedance Magnitude and Phase

In order to measure the impedance of a circuit or circuit element we need to determine the magnitude and phase relationship of the input voltage and input current. This measurement can be accomplished using a dual channel oscilloscope.

Since the oscilloscope is a voltage measurement device, how can we use the oscilloscope to measure current? One way is to determine the voltage across a known circuit element (usually a resistor for convenience) and then to calculate the current via Ohm's law. Measuring the magnitude of voltages and currents is simple: just read the waveform amplitude off the scope display. Measuring the phase relationship is a bit more complicated since we need to see the time relationship between two waveforms. There are several methods to measure phase; two of them are described below.

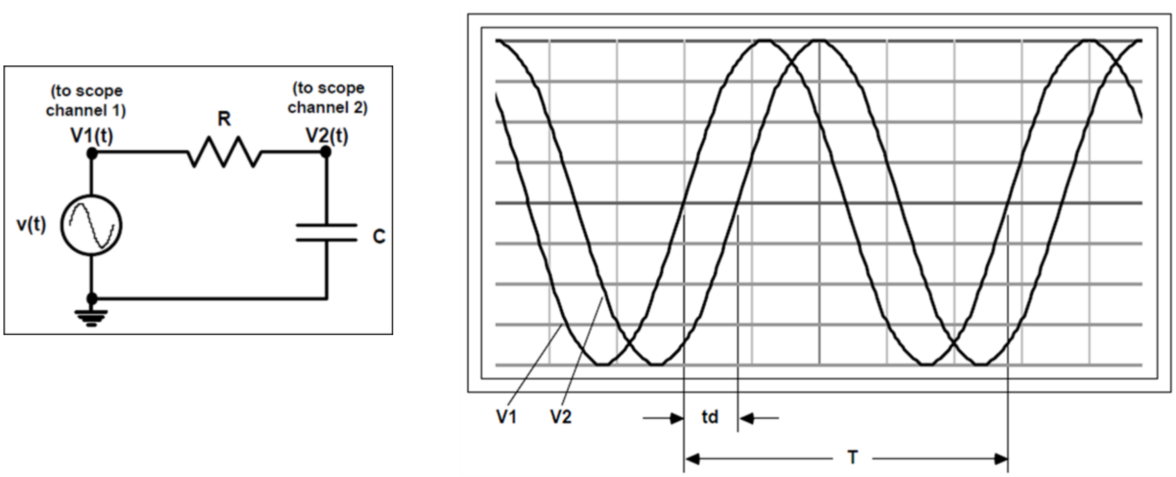

Figure 1: Phase measurement using time difference

- *Phase measurement using time difference*

One way to measure the phase difference between two signals with the same frequency is to set the scope for dual trace operation and observe the time difference between the reference waveform and the waveform measured at the desired node in the circuit. This is depicted in Figure 1.

The phase difference between the two voltages is $2\pi \cdot (td/T)$ radians, or $360° \cdot (td/T)$ degrees.

- *Phase measurement using Lissajous figures*

Another way to determine the phase difference between two sinusoids is to make use of the X-Y capability of the oscilloscope. In this technique, the reference sinusoid is applied to the horizontal (X) deflection of the scope display while the other sinusoid is applied to the vertical (Y) deflection. Since the two waveforms have the same frequency but possibly differing phases, the resulting display will be an ellipse. For example, if the two waveforms are in phase they will both reach their maxima and minima at the same time, giving a straight diagonal line (a "squashed" ellipse). If the two waveforms are 90° out of phase, the maxima and minima of one waveform always occur when the other waveform is zero, giving an ellipse with its primary axis running either horizontally or vertically depending upon the amplitude relationship of the two signals. Other phase differences give elliptical displays with various orientations. These patterns, named in honor of the French mathematician Lissajous, are known as Lissajous figures.

The phase difference φ between the two signals can be determined as follows.

Let:

$$x = A \sin \omega t \text{ and } y = B \sin(\omega t + \varphi),$$

and from Figure 2 we see that when $x = 0$, $|y| = B \sin \varphi = Yo$, (i.e., when $x = 0$, ωt must be a multiple of π).

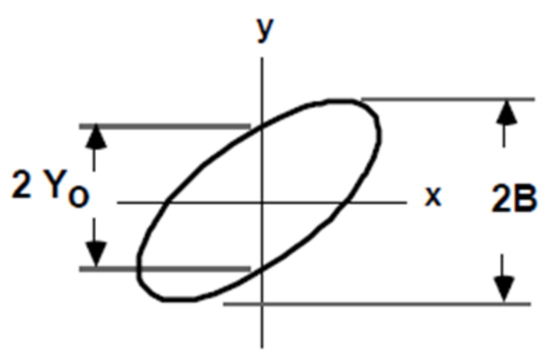

Figure 2: Lissajous figure

Therefore, the phase difference is given by:

$$\sin\varphi = Yo/B \text{ or } \varphi = \sin^{-1}(Yo/B)$$

Note: If the phase difference exceeds 90° the Lissajous ellipse is tilted the other way the formula must be modified to $\varphi = 180° - \sin^{-1}(Yo/B)$.

3. Experiment

(1) Construct the series RC circuit of Figure 1. Use the function generator, a 220Ω resistor and a 0.1μF capacitor. Remember to measure and record the actual values of the components used.

Choose an appropriate ac signal amplitude, Vm (large enough to display easily with the scope but within the power limitations of the resistor used). Setup the scope so that measurements of the input voltage and current can be made (or calculated) easily.

Determine the magnitude and phase of the series RC impedance over at least the frequency range 200Hz through 200kHz. Note that you can determine the current from measurements across the resistor using Ohm's law. Try a few of the measurements with each of the phase determination methods described previously to see which technique seems most convenient and reliable. Determine the impedance at enough different frequencies to determine the behavior of the impedance magnitude and phase. Be sure to verify the signal frequency with the oscilloscope (the markings on the function generator dial may not be extremely accurate). Also, be sure to measure the function generator output at each frequency since the output amplitude from the generator may be different at different frequencies!

Typically, the impedance-vs.-frequency data is plotted on a logarithmic frequency scale (base-10). A good starting point is to measure and plot the data at an approximately equally-spaced log scale, such as 200Hz, 500Hz, 1kHz, 2kHz, 5kHz, 10kHz, etc. Note that the "1, 2, 5" spacing results in roughly equal base-10 logarithmic steps. Be sure to take additional measurements if the data plot turns up any "interesting" features (bends, peaks, etc.).

(2) Now replace the nominal 220Ω resistor with a 1kΩ resistor and again determine the impedance over the same frequency range as before.

4. Results

(a) Prepare a plot of the impedance phase measurements made in part 1 of the experiment. Show the data as a semi-log plot: phase in degrees (linear scale) versus frequency (logarithmic scale). On the same plot, show the mathematical prediction for the phase, (i.e., determine the impedance of the series RC circuit). Discuss the results. Which of the phase measurements seemed to work the best for you? Why?

(b) Prepare two plots of the impedance magnitude measurements made in part 1. For the first plot, show the magnitude measurements along with the mathematical prediction as a semi-log plot: magnitude in ohms (linear scale) versus frequency (log scale). For the second plot, again show the measured and calculated magnitude values but use a log-log plot: magnitude (log scale) versus frequency (log scale). Discuss the results and compare the features of the two data plots.

(c) Prepare a plot of the impedance phase measurements (semi-log) and a plot of the impedance magnitude measurements (log-log) from part 2. On the same plots show the mathematical predictions for the impedance. Discuss the differences between the plots for part 1 and the plots for part 2.

(d) What would you change about the procedures of this experiment?