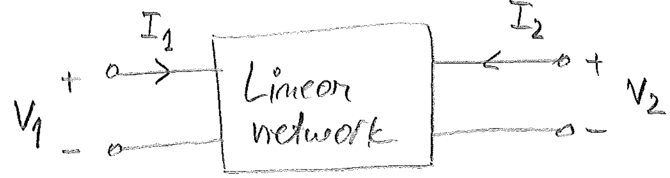
Chapter 17 Two-port Networks

1 Admittance parameters (also known as "the y parameters")



NOTE: For reasons of symmetry, it is convenient to assume the passive-sign convention for both ports!

$$\begin{cases} I_1 = y_{11} V_1 + y_{12} V_2 \\ I_2 = y_{21} V_1 + y_{22} V_2 \end{cases}$$

Matrix format:

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{\triangleq I} = \underbrace{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}}_{\triangleq y} \cdot \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{\triangleq V} \Rightarrow \boxed{I = y \cdot V}$$

- y_{11} is the admittance measured at the input port with the terminals of the output port short-circuited ($V_2 = 0$)

(1) $$\boxed{y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}} \triangleq \text{" short-circuit input admittance "}$$

(2) $$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

(3) $$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

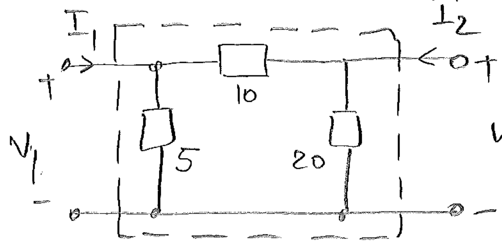
(2), (3) $\triangleq$ "short-circuit transfer admittance"

(4) $$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \triangleq \text{"short-circuit output admittance"}$$

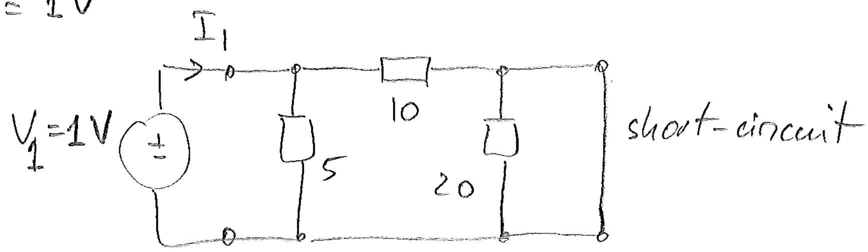
All in [S] Siemens

NOTE: It is easier to use equations (1) ÷ (4) when only one parameter is desired! If we need all of them, it's easier to assign V_1, V_2 and solve the circuit with known techniques!

Example Find "y parameters."



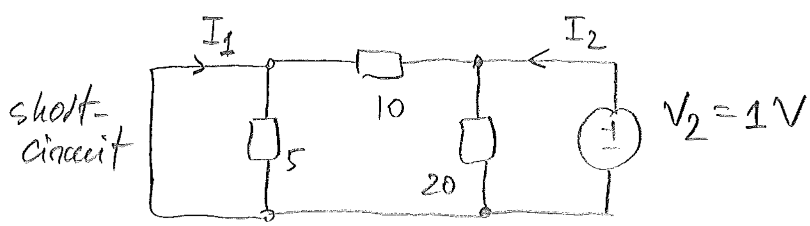
- To determine y_{11}, we short-circuit the output and find the ratio of I_1 and V_1 as shown in equation (1).
- Let $V_1 = 1V$



$$V_1 = I_1 \cdot (5 \parallel 10) = I_1 \cdot \frac{5 \cdot 10}{5+10}$$

$$\Rightarrow y_{11} = \frac{I_1}{V_1} = \frac{15}{50} = 0.3 \ S \qquad \boxed{y_{11} = 0.3 \ S}$$

- To determine y_{12}, we short-circuit the input terminals and apply 1V at the output terminals:



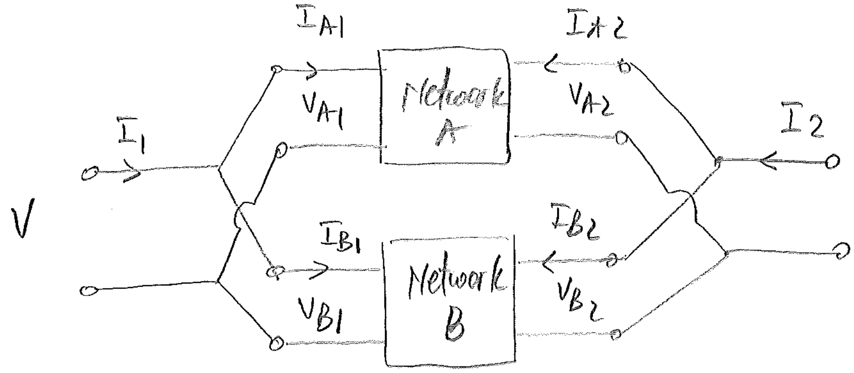
$$I_2 = \frac{V_2}{10 \parallel 20} = \frac{10+20}{10 \cdot 20} = \frac{30}{200} = \frac{3}{20} \ A$$

$$I_1 = -\frac{20}{20+10} \cdot I_2 = -\frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{20} A = -\frac{1}{10} A$$

$$\Rightarrow y_{12} = \frac{I}{V_2} = \frac{-\frac{1}{10}}{1} = -0.1 \ S \qquad \boxed{y_{12} = -0.1 \ S}$$

- By similar methods we can compute: $\boxed{y_{21} = -0.1 \ S}$; $\boxed{y_{22} = 0.15 \ S}$

- y parameters are useful in describing the parallel connection of two-ports!



- Network A:

$$\boxed{I_A = y_A V_A} \quad \text{where} \quad I_A = \begin{bmatrix} I_{A1} \\ I_{A2} \end{bmatrix} \quad V_A = \begin{bmatrix} V_{A1} \\ V_{A2} \end{bmatrix}$$

- Network B:

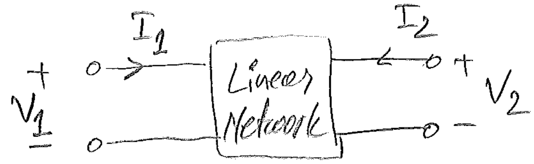
$$\boxed{I_B = y_B V_B} \quad \text{where} \quad I_B = \begin{bmatrix} I_{B1} \\ I_{B2} \end{bmatrix} \quad V_B = \begin{bmatrix} V_{B1} \\ V_{B2} \end{bmatrix}$$

- Note: $V_A = V_B = V \qquad I = I_A + I_B$

$$\Rightarrow \boxed{I = \underbrace{(y_A + y_B)}_{\triangleq y} V = y \cdot V}$$

- Each $\boxed{y}$ parameter of the parallel network is given as the $\boxed{\text{sum}}$ of the corresponding parameters of the individual networks!

2 Impedance parameters (the "z parameters)



$$\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

Matrix format:

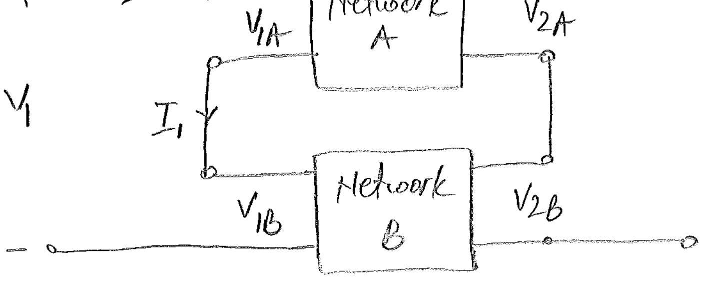
$$\underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{\triangleq V} = \underbrace{\begin{bmatrix} z_{11} & z_{12} \\ z_{22} & z_{22} \end{bmatrix}}_{\triangleq z} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{\triangleq I} \Rightarrow \boxed{V = z \cdot I}$$

NOTE: It is not necessary that I_1, I_2 or V_1, V_2 be current or voltage sources. They may have any networks terminating the two-port at either end!

$$z_{11} = \frac{V_1}{I_1}\Big|_{\underbrace{I_2=0}} \qquad z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

open circuit at output terminals

- These are "open-circuit impedance parameters"
- z parameters are useful to simplify the problem of a series connection of networks!



Series connection

$I = I_A = I_B$

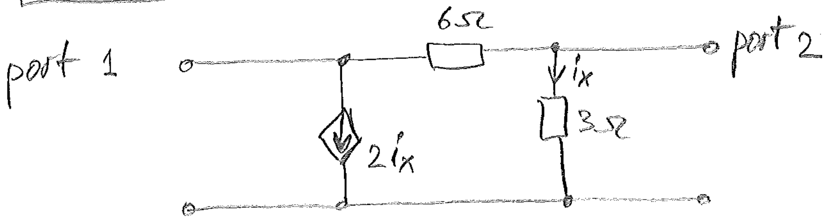
$V = V_A + V_B = z_A I_A + z_B I_B \Rightarrow$

$$V = \underbrace{(z_A + z_B)}\, I$$

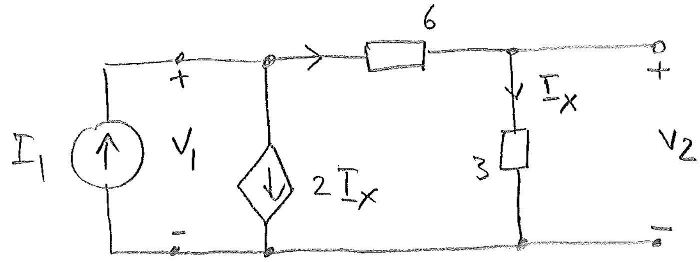
$$\boxed{V = z \cdot I}$$

where: $\boxed{z = z_A + z_B}$

$\boxed{\text{Example:}}$ Find $[z]$ for the circuit:



- To find z_{11} and z_{21}, apply a test current I_1 to port 1 and leave port 2 open-circuited to ensure $I_2 = 0$.



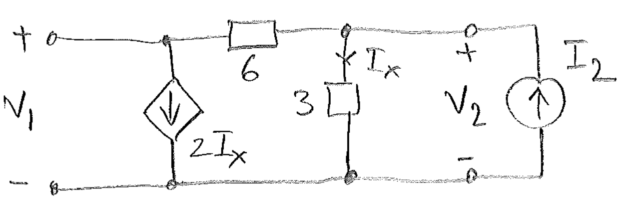
$$I_x = \frac{V_2}{3}$$

$$\begin{cases} I_1 = 2I_x + \dfrac{V_1 - V_2}{6} = \dfrac{2}{3}V_2 + \dfrac{V_1 - V_2}{6} \\ \dfrac{V_1 - V_2}{6} = I_x = \dfrac{V_2}{3} \end{cases}$$

$\Rightarrow$ Solve for V_1, V_2 in terms of $I_1 \Rightarrow$

$$\Rightarrow \begin{cases} V_1 = 3I_1 \\ V_2 = 1 \cdot I_1 \end{cases} \Rightarrow \begin{matrix} \boxed{z_{11} = 3\,\Omega} & (1) \\ \boxed{z_{21} = 1\,\Omega} & (2) \end{matrix}$$

- To find z_{12} and z_{22}, apply a test current I_2 to port 2 and leave port 1 open-circuited to ensure $I_1 = 0$.



$$I_x = \frac{V_2}{3}$$

$$\begin{cases} I_2 = I_x + \dfrac{V_2 - V_1}{6} = \dfrac{V_2}{3} + \dfrac{V_2 - V_1}{6} \\ \dfrac{V_2 - V_1}{6} = 2I_x = \dfrac{2}{3}V_3 \end{cases}$$

$\Rightarrow$ Solve to get:

$$\begin{cases} V_1 = -3I_2 \\ V_2 = 1\,I_2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{matrix} \boxed{z_{12} = -3\,\Omega} & (3) \\ \boxed{z_{22} = 1\,\Omega} & (4) \end{matrix} \quad \xrightarrow[(2)]{(1)} \quad \boxed{\text{Finally: } [z] = \begin{bmatrix} 3 & -3 \\ 1 & 1 \end{bmatrix} \Omega}$$