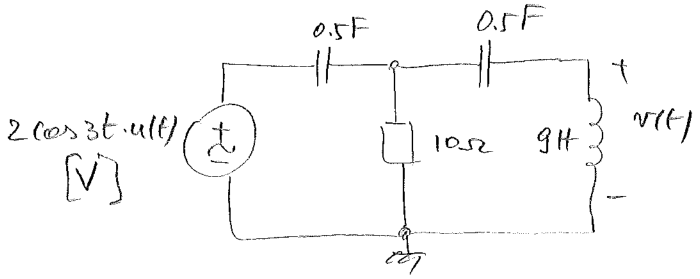
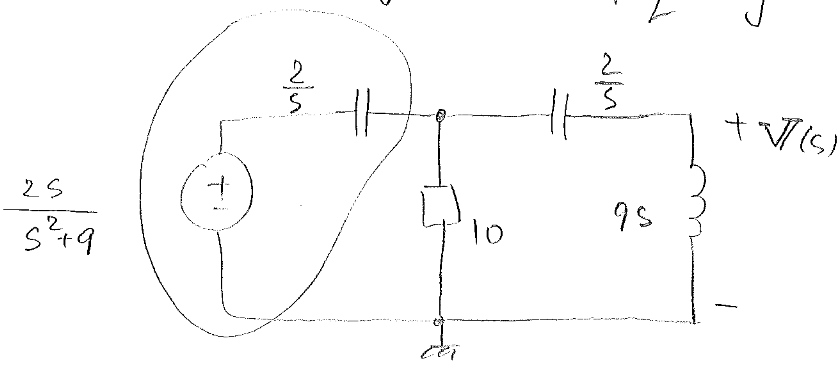
3 Source transformations

Example 1: Use source transformations to find an expression for $v(t)$

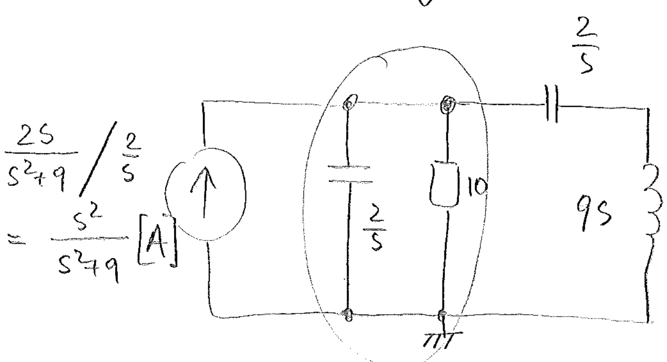


Observe that there are no initial conditions!

⇓ convert to frequency s-domain

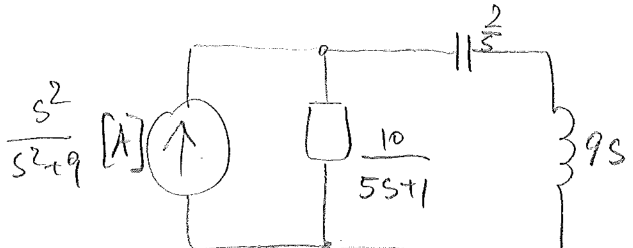


⇓



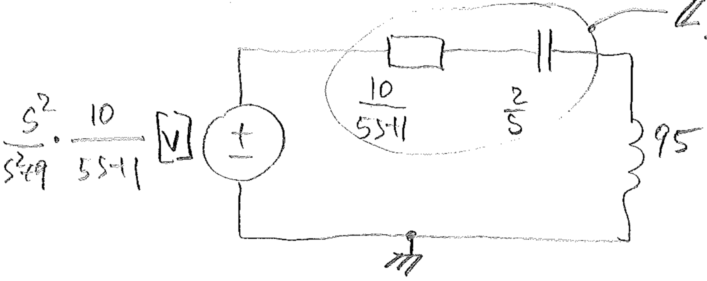
$$Z_1 = \frac{2}{s} \,\|\, 10 = \frac{\frac{20}{s}}{\frac{2}{s}+10} = \frac{20}{10s+2} = \frac{10}{5s+1} \quad [\Omega]$$

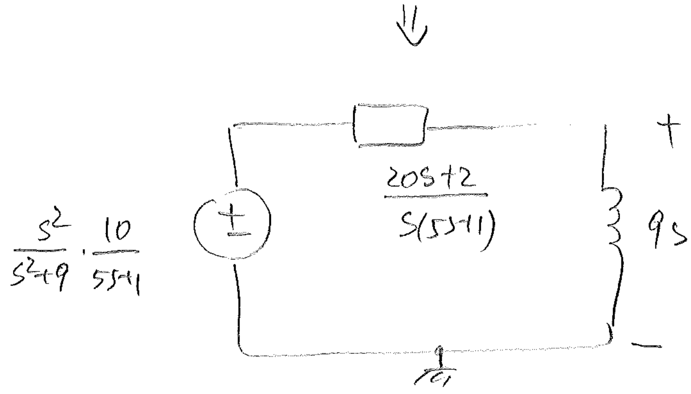
⇓



⇓

$$Z_2 = \frac{10}{5s+1} + \frac{2}{s} = \frac{20s+2}{s(5s+1)} \quad [\Omega]$$





By voltage division:

$$\mathbb{V}(s) = \frac{9s}{9s + \frac{20s+2}{s(5s+1)}} \times \frac{s^2}{s^2+9}\cdot\frac{10}{5s+1}$$

$$= \frac{180\,s^4}{(s^2+9)(90s^3+118s^2+40s+4)}$$

$$\mathbb{V}(s) = \frac{A_1}{s-j3} + \frac{A_2}{s+j3}^{=A_1^*} + \frac{A_3}{s+0.05-j0.66} + \frac{A_4}{s+0.05+j0.66}^{=A_3^*} + \frac{A_5}{s+0.1}$$

To compute A_1 and A_3, see <u>lecture notes</u> of week #5, Wed.:

complex
$$\begin{cases} A_1 = (s-j3)\times \mathbb{V}(s)\Big|_{s=j3} \cong 1.05 + j0.07 \\ A_2 = A_1^* = 1.05 - j0.07 \\ A_3 = (s+0.05-j0.66)\times \mathbb{V}(s)\Big|_{s=-0.05+j0.66} \cong -0.05 - j0.02 \\ A_4 = A_3^* = -0.05 + j0.02 \end{cases}$$

Real: $A_5 = (s+0.1)\times \mathbb{V}(s)\Big|_{s=-0.1} \cong 5.6\times 10^{-5}$

- <u>Hence</u>:

$$\mathbb{V}(s) = \frac{1.05+j0.07}{s-j3} + \frac{1.05-j0.07}{s+j3} - \frac{0.05+j0.02}{s+0.05-j0.66} - \frac{0.05-j0.02}{s+0.05+j0.66} + \frac{5.6\times 10^{-5}}{s+0.1}$$

Finally, see lecture notes of week #5, Wed:

$$v(t) = \mathcal{L}^{-1}\{V(s)\} =$$

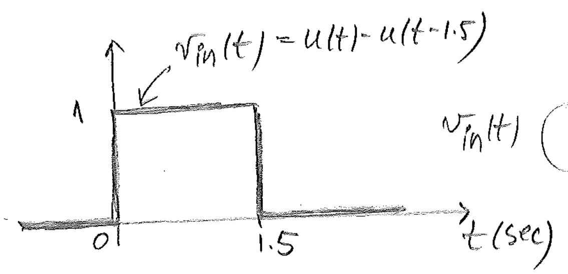
$$= 5.6\times10^{-5} \cdot e^{-0.1t} \cdot u(t) +$$

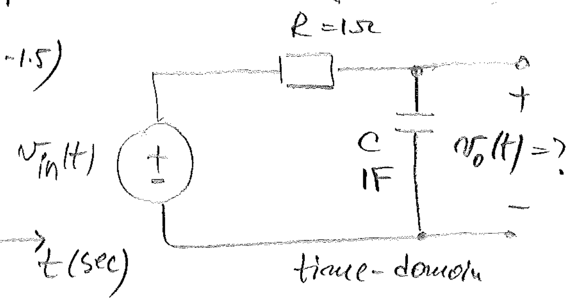
$$2.1 \cdot \cos(3t + 3.91^\circ) \cdot u(t)$$

$$0.1 \cdot e^{-0.05} \cdot \cos(0.66t + 157.9^\circ) \cdot u(t)$$

$\boxed{\text{Example 2:}}$

Find the response of circuit to signal $v_{in}(t)$.





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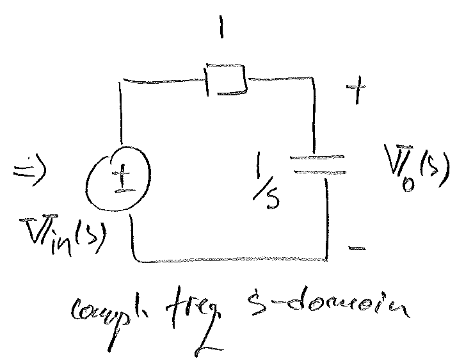

- observe that initial conditions are zero.
- Find $\boxed{\text{network function}}$ ("output over input"): use voltage division:
  or $\boxed{\text{transfer function}}$

$$H(s) = \frac{V_0(s)}{V_{in}(s)} = \frac{1/s}{1 + 1/s} = \frac{1}{s+1}$$

- Therefore, the output or the response $V(s)$ can be computed as:
  (see lecture notes week #7, Wed):

$$V_0(s) = H(s) \cdot V_{in}(s) + \underbrace{V_{0,natural}(s)}_{=0}$$

=0 because there are no initial conditions.

$$H(s) \rightarrow \frac{1}{s+1} \qquad V_{in}(s) \rightarrow \frac{1}{s} - e^{-1.5s}\cdot\frac{1}{s} = \frac{1}{s}\left(1 - e^{-1.5s}\right)$$

$$V_0(s) = \frac{1}{s+1} \times \frac{1}{s}\left(1 - e^{-1.5s}\right) = \frac{1}{s(s+1)} \cdot \left(1 - e^{-1.5s}\right) =$$

$$= \left[\frac{1}{s} - \frac{1}{s+1}\right] \cdot \left(1 - e^{-1.5s}\right)$$

$$= \left(\frac{1}{s} - \frac{1}{s+1}\right) - \left(\frac{1}{s} - \frac{1}{s+1}\right) e^{-1.5s}$$

- Finally:

$$v_0(t) = \mathcal{L}^{-1}\left\{ V_0(s) \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s} - \frac{1}{s+1} \right\} - \mathcal{L}^{-1}\left\{ \left(\frac{1}{s} - \frac{1}{s+1}\right) e^{-1.5s} \right\}$$

$$= \left(1 - e^{-t}\right) \cdot u(t) - \left[1 - e^{-(t-1.5)}\right] \cdot u(t-1.5)$$