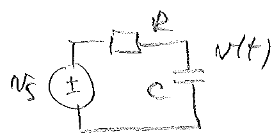
Review for exam #3

Not given

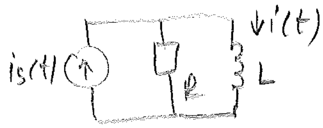
Notes:

- I expect you to know that:

$\tau = RC$



$\tau = \frac{L}{R}$



- You are expected to know that when $x(t) = 0$ the response (natural) is:

$$\boxed{y(t) = y(0)\, e^{-\frac{t}{\tau}}}$$

Given:

$$\boxed{\tau \frac{dy(t)}{dt} + y(t) = x(t)}$$

Constant forcing-function:

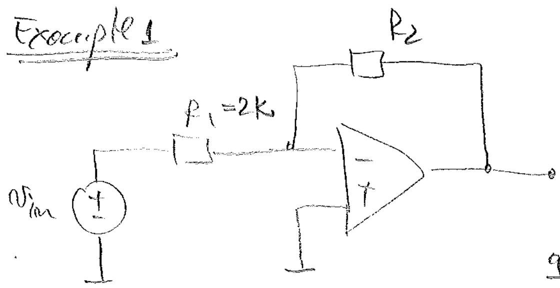
$$\boxed{x(t) = X_S}$$

$$\boxed{y(t) = [y(0) - y(\infty)]\, e^{-\frac{t}{\tau}} + y(\infty)}$$

$$\boxed{x(t) = X_m \cdot \cos \omega t}$$

$$y(t) = \left[y(0) - \frac{X_m}{1+(\omega\tau)^2}\right] e^{-\frac{t}{\tau}} + \frac{X_m}{\sqrt{1+(\omega\tau)^2}} \cdot \cos\left(\omega t - \tan^{-1}\omega\tau\right)$$

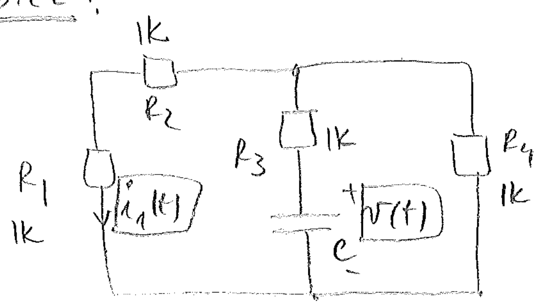
Example 1



$V_{out} = V_{out}(v_{in}) = ?$

$R_2 = ?$ such that $|\text{gain}| = 3$

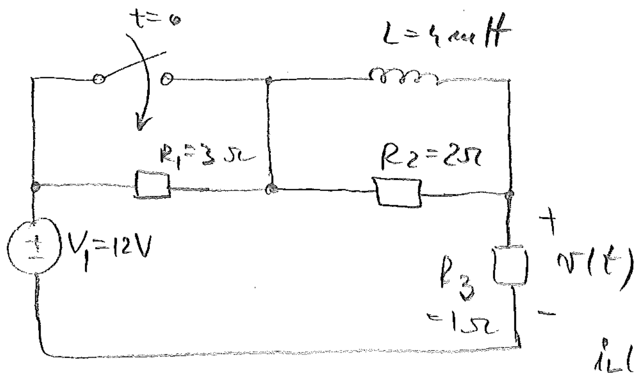
Example 2:



$V_0 = v(0^-) = 15V$; $v(t) = ?$; $i_1(t) = ?$

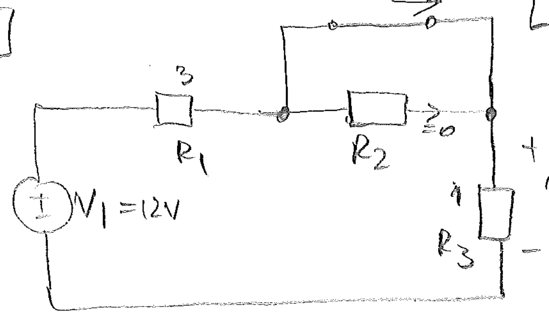
Example 3:

(a) Sketch and label $v(t)$; (b) Find the instant when $v(t) = 10V$!



(a) $\boxed{t<0}$

$i_L(0^-)=3A$ — L is short in dc circuits let for long time!

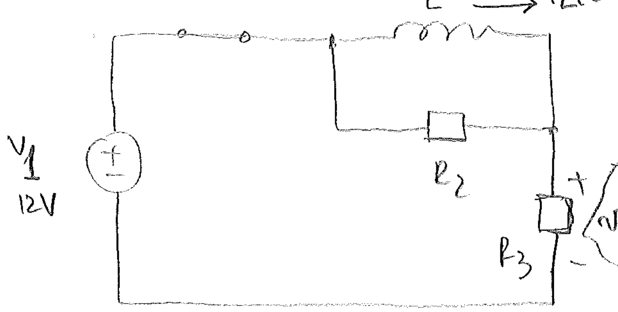


$$v(0^-) = \frac{R_3}{R_1+R_3} \cdot V_1 = \frac{1}{1+3} \cdot 12 = 3V$$

Also: $i_L(0^-) = \dfrac{V_1}{R_1+R_3} = \dfrac{12}{1+3}A = 3A = i_{R_3}(0^-)$

$\boxed{t \geq 0}$

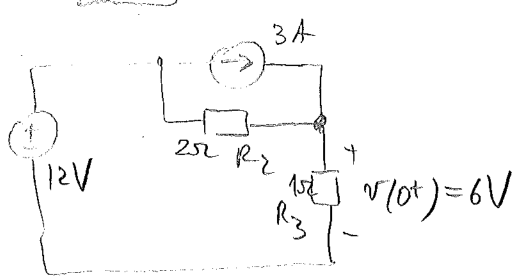
$L \longrightarrow i_L(0^+) = i_L(0^-) = 3A$



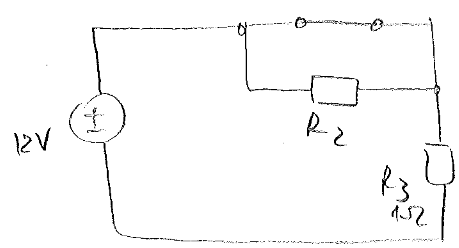
$$v(t) = \left[v(0^+) - v(\infty)\right] e^{-\frac{t}{\tau}} + v(\infty)$$

form expected.

▶ For $\boxed{t=0}$



▶ For $\boxed{t=\infty}$



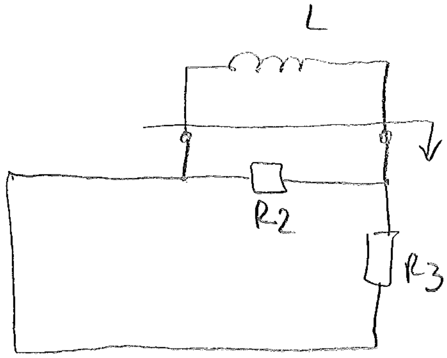
Use superposition:

$$v(0^+) = \frac{R_3}{R_3+R_2} \times 12V + (R_2 \| R_3) \times 3A$$

$$= \frac{1}{2+1} \times 12 + \frac{1 \cdot 2}{2+1} \times 3$$

$$= 6V$$

we still need: $\tau = \frac{L}{R}$, $R \triangleq$ resistance "seen" by L at its terminals.

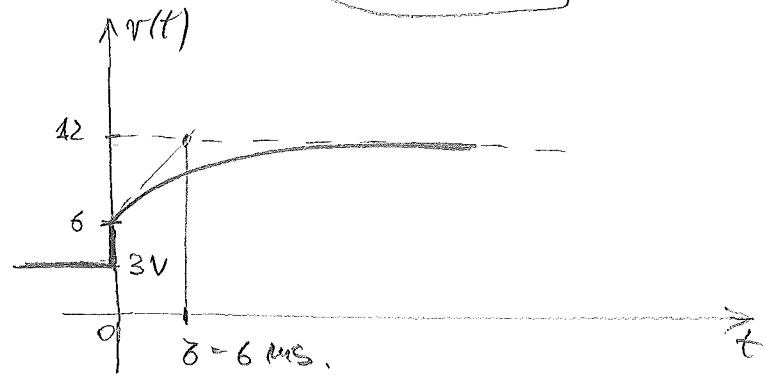


$R_0 = R_2 \| R_3 = 1 \| 2 = \frac{2}{3}\,\Omega$

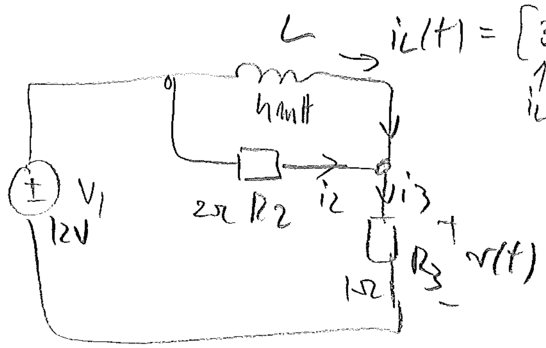
$$\tau = \frac{L}{R} = \frac{4\,mH}{\frac{2}{3}} = 6\,ms$$

Hence complete response/solution for $v(t)$:

$$\boxed{v(t) = [6 - 12]\,e^{-\frac{t}{6ms}} + 12} = \boxed{12 - 6\,e^{-\frac{t}{6ms}}} \quad ; t \geq 0.$$



Verificare: (skip this) (Method 2)



$i_L(t) = [3 - 12]\,e^{-\frac{t}{\tau}} + 12 = 12 - 9\,e^{-\frac{t}{6}}$ $\qquad \tau = \frac{L}{R_0}$

$i_L(0^+)$

$v_L(t) = L\frac{di}{dt} = \cancel{L}\cdot\frac{R}{\cancel{L}}\cdot 9\,e^{-\frac{t}{6}} = \frac{2}{\cancel{3}}\cdot \overset{3}{\cancel{9}}\,e^{-\frac{t}{6}} = 6\,e^{-\frac{t}{6}}$

$i_2 = \frac{v_L(t)}{R_2} = 3\,e^{-\frac{t}{6}}$

$i_L(t) + i_2(t) = i_3(t)$

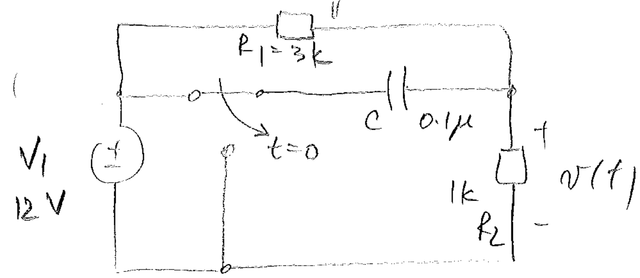
$12 - 9\,e^{-\frac{t}{6}} + 3\,e^{-\frac{t}{6}} = i_3(t) = 12 - 6\,e^{-\frac{t}{6}}$

$$\boxed{v(t) = R_3 \cdot i_3(t) = 12 - 6\,e^{-\frac{t}{6}}} \quad ; t \geq 0$$

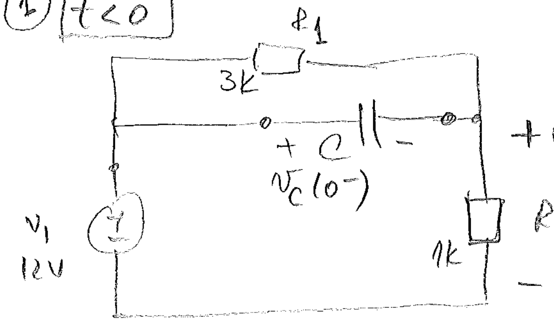
same result!

Example (4)

Sketch the voltage $v(t)$



① $t<0$

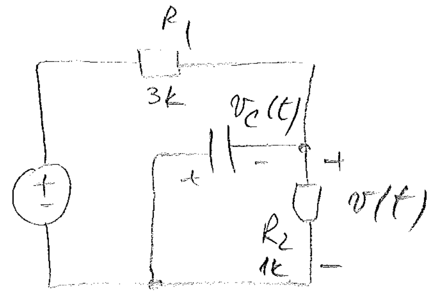


after very long transients have died out, this is a dc circuit with C an open circuit!

$$\boxed{v(0^-) = \frac{R_2}{R_2+R_1} V_1 = \frac{1}{3+1} \times 12\,V = 3\,V}$$

By KVL: $\boxed{v_C(0^-) = V_1 - v(0^-) = (12-3)\,V = 9\,V}$

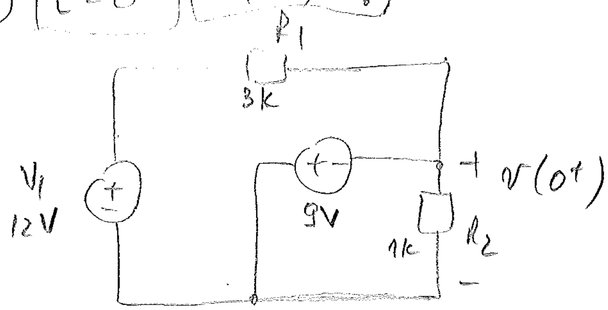
② $t \geq 0$ →



We expect:

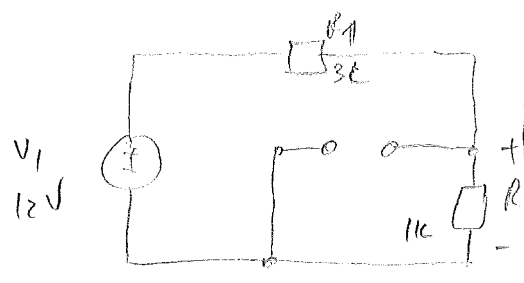
$$\boxed{v(t) = [v(0^+) - v(\infty)]\, e^{-\frac{t}{\tau}} + v(\infty)}$$

ⓐ $t = 0^+$ $v(0^+) = ?$



By KVL we have $\boxed{v(0^+) = -9\,V}$

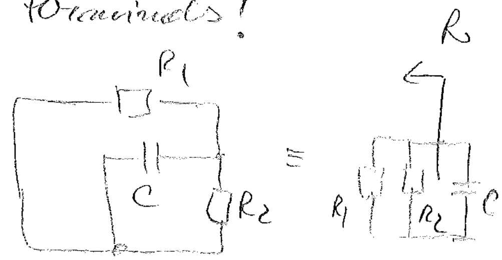
ⓑ $t = \infty$ $v(\infty) = ?$



$$\boxed{v(\infty) = \frac{R_2}{R_1+R_2} V_1 = 3\,V}$$

ⓒ $\tau = RC = \frac{3}{4} k \times 0.1\mu F = \frac{0.3}{4}\, ms$

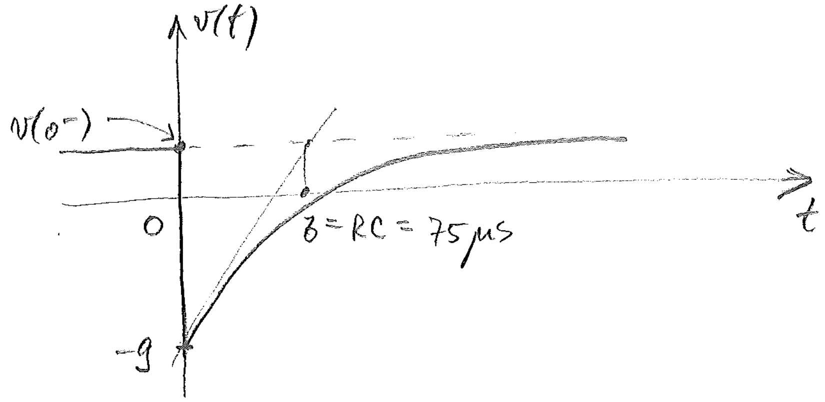
resistance seen by C at its terminals!



$R = R_1 \| R_2 = 1 \| 3 = \frac{1 \times 3}{1+3} = \frac{3}{4}\, k$

Hence:

$$v(t) = [-9-3]\, e^{-\frac{t}{\tau}} + 3 = \boxed{3 - 12\, e^{-\frac{t}{\tau}} = v(t)} \quad t \geq 0$$



v(t)
v(0⁻)
0
τ = RC = 75 μs
-9
t